# 2018 John O'Bryan Mathematical Competition
# Freshman-Sophomore Individual Test

**Directions:** Please answer all questions on the answer sheet provided. All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. **Exact** answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. Find the largest odd three-digit number such that the hundreds digit is equal to twice the ones digit and the tens digit is even.

2. The expressions $a(x-2)^2 + (3x+1)^2$ and $(2x-5)^2 + 12(x^2 + kx) + c$ are equivalent for all values of $x$. Determine the product $(akc)$. Express your answer as a common or improper fraction reduced to lowest terms.

3. An odd number and an even number are picked at random from the first ten positive integers. Find the probability that their sum is 11. Express your answer as a common fraction reduced to lowest terms.

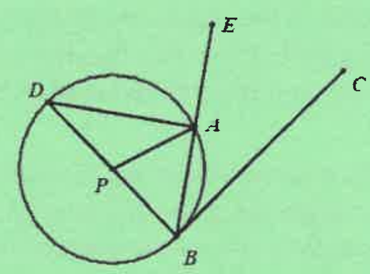
4. Define a "mid-diagonal" to be a segment connecting a midpoint of one side of a polygon to a vertex of the polygon other than the two vertices that are endpoints of the side on which the midpoint is located. Find the number of mid-diagonals of an octagon.

5. The points $(6,3)$, $(-1,2)$, and $(9,k)$ are collinear with $k = \frac{a}{b}$ where $a$ and $b$ are relatively prime positive integers. Find the value of $\sqrt{a^2 + b^2}$.

6. The ratio *width:length:height* of a rectangular solid is 8:15:144. If the length of a diagonal of the rectangular solid is 108.75, find the width of the solid.

7. In the following number base problem, if $\frac{1}{6}$ of $40_x$ is 8 (base 10), then find $\frac{1}{7}$ of $36_x$. Express your answer as a base 10 number, written without the base.

8. Given circle $P$ with tangent $\overline{BC}$, secant $\overline{BE}$, and diameter $\overline{DB}$ (as showin in the diagram). If the measure of $\angle ABC = 35°$, find the number of degrees in the measure of the minor arc connecting $A$ and $D$.

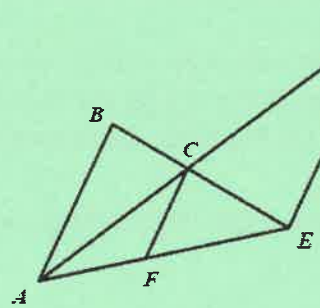



9. Find the smallest positive integer greater than 2018 that leaves the same positive integral remainder when divided by 30, 36, and 50

10. In $\Delta MTH$ the measure of $\angle M$ is $\frac{1}{3}$ the measure of $\angle H$ and $\frac{1}{2}$ the measure of $\angle T$. $MH = a\sqrt{b}$, with $a$ and $b$ being integers, and the sum of the lengths of sides $\overline{MT}$ and $\overline{TH}$ is an integer less than 30. If $b$ is as small as possible, find the sum of all possible values of $a$.

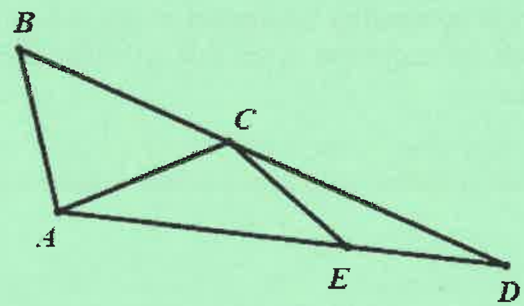
11. Sami rides her bike 3 miles each day to school. The first mile is uphill and she rides at a constant rate of 4 mph. The second mile is downhill, where she coasts at a constant rate that is an integer, is faster than her uphill rate, and is less than 60 mph. For the final mile she rides at a constant rate of 6 mph. Her average speed, in mph, for the entire trip to school is an integer. Find the speed, in mph, at which she coasts down the hill.

12. In the diagram, $\overline{AB} \parallel \overline{CF} \parallel \overline{DE}$ where $AB = 16$ and $DE = 24$, find the length of $CF$. Express your answer as an exact decimal.




13. Circles of radii 4, 5, and 6 are mutually tangent externally. Find the area of the triangle formed by connecting the centers of the three circles. Express your answer in the form $a\sqrt{b}$ where both $a$ and $b$ are positive integers and $b$ is as small as possible.

14. In the diagram, $\overline{BA} \cong \overline{AC} \cong \overline{CE} \cong \overline{ED}$ and $\angle DAB = 110°$. Find the degree measure of $\angle CED$.




15. The number 5760 can be expressed as $4^a 5^b 6^c$ where $a$, $b$, and $c$ are rational numbers. Find the value of $a^4 b^5 c^6$.

16. A triangle is selected at random from the set of triangles with internal degree measure angles for which the ratio of the degree measures of the angles is $a$:$b$:$c$, where $a$, $b$, and $c$ are consecutive positive integers. Find the probability that the degree measure of the largest angle is an odd number. Express your answer as a common fraction reduced to lowest terms.

17. Two triangles are similar but not congruent. The lengths of all sides of both triangles are integers. The first triangle has sides of 9 and 32 and a perimeter of 68. If one of the sides of the second triangle is the same length as one of the sides of the first triangle, find the perimeter of the second triangle.

18. If $x + y = 7$ and $\frac{1}{x} + \frac{1}{y} = \frac{3}{8}$ with $xy \neq 0$, find the value of $(x^2 + xy + y^2)$. Express your answer as a common or improper fraction reduced to lowest terms.

19. On the planet Vulcan, there are 11 major cities. Each pair of cities is connected by exactly one road. There are no other roads on the planet. Find the number of roads on Vulcan.

20. A rectangular sheet of paper 40 cm by 70 cm is folded once so that exactly two of the opposing corners are coincident. Find the number of cm in the length of the crease in the paper. Give your answer as a decimal rounded to the nearest hundredth.

Name: ______ANSWERS______ Team Code: _______

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1. 683
2. $-\dfrac{14}{3}$ Must be this reduced fraction.
3. $\dfrac{1}{5}$ Must be this reduced fraction.
4. 48
5. 25
6. 6
7. 6
8. 110 (degrees optional)
9. 2701
10. 45
11. 12 (mph optional)
12. 9.6 Must be this exact decimal.
13. $30\sqrt{2}$ Must be this exact answer.
14. 145 (degrees optional)
15. 2500
16. $\dfrac{4}{11}$ Must be this reduced fraction.
17. 204
18. $\dfrac{91}{3}$ Must be this reduced fraction.
19. 55
20. 46.07 Must be this exact decimal.

Awards Lists and Solutions to the Team Competition may be found at http://math.nku.edu/job